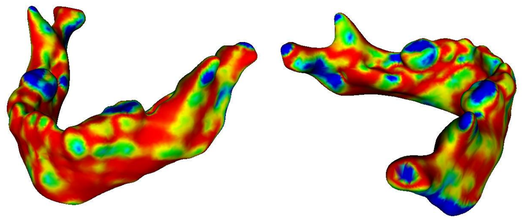
Mathematical Modelling of Mandibular Metamorphosis



Red colors indicate high symmetry weight.

Master-thesis project by Mark Wrobel. Supervisors: Associate Professor, M.Sc.(Eng), Ph.D Rasmus Larsen , Postdoc, Ph.D., M.Sc. Klaus Baggesen Hilger.

Modelling mandibular growth

A growth model of the mandible will give better insight to the understanding of facial growth. Especially when performing pediatric craniofacial surgery, knowledge of growth is of paramount importence.
Former studies have shown, that it is possible to construct a growth model that allows for prediction of mandible size and shape from scans of only 1-3 month old patients by the use of PCA (Principal Component Analysis).

The mandible dataset

The dataset consists of 6 patients each of which has been longitudinally CT-scanned between three and seven times. The dataset is presented below.

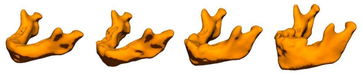
Patient 1

Patient 2

Patient 3

Patient 4

Patient 5

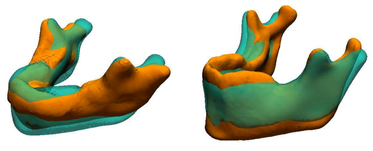


Patient 6

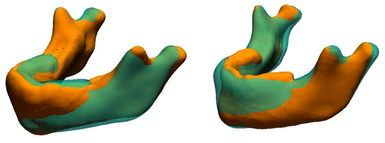
Patient 1-5 originates from St. Louis Children's Hospital, USA, and patient 6 comes from the University Hospital of Denmark. Each shape consists of 14851 landmarks and there is a unique correspondence between landmarks across shapes. Hence the dataset lend itself easily to statistical analysis and decomposition.

Principal Component Analysis

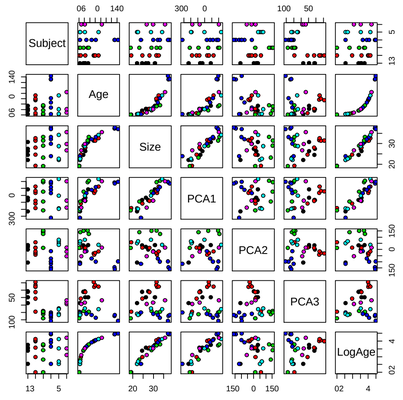
The PCA can be used to extract a subspace of the data-space. The eigenvectors of this sub-space can then be used to model growth.
The figure below shows what happens when we estimate our shape using the first and second principal component.
The cyan color corresponds to the mean shape.



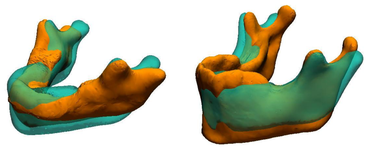
PCA1 –2 std. PCA1 +2 std.



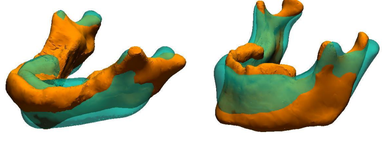
PCA2 –2 std. PCA2 –2 std.



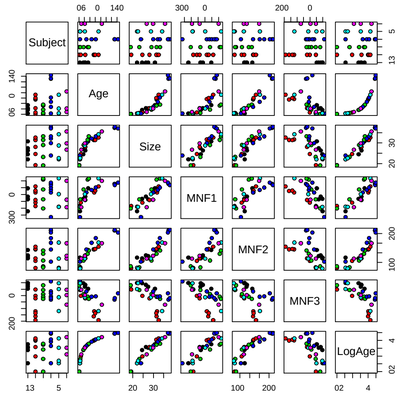
Minimum Noise Fractions



MNF1 +2 std. MNF1 -2 std.



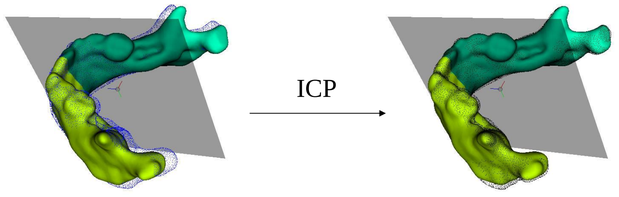
MNF2 +2 std. MNF2 –2 std.



MNF is another method to decompose the dataspace. PCA just maximize variance e.g. the first component has the largest variance orthogonal to all other components. MNF on the other hand maximizes signal-to-noise ratio.

Defining weights based on symmetry

We use the assumption that the mandible is a perfect symmetrical bone. Especially for younger specimens this seems to be a fair assumption. However, when we look at the somewhat older specimens the assumption seems to fail because of the asymmetric eruption of teeth's.
Since we are primarily interested in separating mandibular growth from tooth eruption, weights based on symmetry seems to be the way to go.



By mirroring the mandible around a plane an initial pointset is given. This pointset is then aligned to the mandible by the Iterative Closest Point algorithm. The closest distance between a point in the pointset and the surface of the mandible can now be calculated, yielding the symmetry weights.

Future work

Make decomposition based on symmetry weights and from this work deduce a growth model etc.

Informatics and Mathematical Modelling

This project is done in cooperation with:
3D-Lab, School of Dentistry, University of Copenhagen.